Exponential Equations and Logarithmic Equations

Consider the problem:

The population of Bongotown is 1025. Every year the population increases by a factor of 1.21.

A. Write an equation modeling the situation. $P = 1025 \cdot 1.21^t$

B. How many people live in the town in 5 years? $P = 1025 \cdot 1.21^5 = 2658$

C. How long until the population increases to 1500? $1500 = 1025 \cdot 1.21^t$

$1.463 = 1.21^t$ → Now What????

To solve this problem and ANY problem like this, we need to create something new to handle the math.

Logarithms are functions which are inverses of exponential equations.

They look like: $log_4 x$ → The little, subscripted number is called the base.

$log_4 x$ is the inverse of the function 4^x.

THE FOLLOWING IS THE MAJOR POINT OF THIS CHAPTER:

Every equation involving exponential functions can be written as an equation involving logarithmic functions.

$y = b^x$	⇔	$x = log_b\, y$
Exponential Equation		Logarithmic Equation

Ex: Write the equation$5 = 17^x$, in LOGARITHMIC form.

$5 = 17^x$ is of the form $y = b^x$ → y = 5, b = 17, x = x → so $x = log_b y$ becomes $x = log_{17} 5$

Ex: Write the equation, $6 = log_2 w$ in EXPONENTIAL form.

$6 = log_2 w$ is of the from $x = log_b y$ → x = 6, b = 2, y = w → so $y = b^x$ becomes $w = 2^6$

Ex: Solve the equation: $8 = log_3 x$ → CHANGE to $x = 3^8$ = 6561

Ex: Solve the equation: $x = log_3 81$ → CHANGE to $81 = 3^x$ → 3 to what power is 81? x = 4

Ex: Solve the equation: $\frac{1}{2} = log_{64} x$ → CHANGE to $x = 64^{\frac{1}{2}} = \sqrt{64} = 8$